Combined Diagram: A Graphical Representation of Combination Evaporation Rates

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ABSTRACT

Combination methods estimate the partition of sensible and latent heat fluxes at the surface by combining the surface energy balance equation with the transfer equations for temperature and water vapor in the atmospheric surface layer. This paper presents a diagram that graphically depicts the main assumptions and steps of the method. Potential, apparent potential, and actual evaporation rates are visually explained with the diagram. Using a linearized saturation water content relation, the diagram permits the derivation of many of the combination evaporation results by means of geometrical analysis. The diagram should be helpful in the teaching and understanding of the combination methodology.

1. Introduction

Water-temperature diagrams, in various forms, help in the understanding of meteorological problems in which temperature and water (including its phase changes) play important roles. Examples begin with traditional thermodynamic diagrams, like the skew $T$–log $p$ or the tephigram, which concisely show the joint vertical structure of temperature and water and are invaluable in the assessment of several types of stability of an atmospheric column (e.g., Bohren and Albrecht 1998). Other water-temperature diagrams have been used to illustrate the temperature and water budgets of the atmospheric boundary layer (ABL) over land (Betts 1992) and their control by soil moisture (Betts and Ball 1995, 1998), the mixing properties of cumulus clouds (Betts 1985; Blyth et al. 1988), the buoyancy of clear–cloudy parcel mixtures (Deardorff 1980), the effect of entrainment on the depth of stratocumulus cloud layers (Randall 1984), etc. Because of the critical role heat and moisture play in all of these processes, methods for visualizing the effects of both of these quantities simultaneously greatly enhance the understanding of the underlying processes.

The surface energy budget (SEB) is another example in which water and temperature processes are intimately intertwined. In particular, the problem of assessing what fraction of the energy available at the surface is exchanged with the atmospheric boundary layer in the form of sensible or latent heat fluxes is of utmost importance in meteorological models as well as in observational analysis. One main class of methods to determine this SEB partition is that of the so-called combination equations, which combine the SEB with the turbulent transfer equations for water vapor and temperature in the atmospheric surface layer (e.g., Garratt 1994). The standard treatment in many textbooks and introductory surveys, however, presents the methodology based mostly on the mathematical equations, followed by figures in which sensitivities of the results upon different factors are directly shown (e.g., Garratt 1994; Brutsaert 2005; Rowntree 1991). To the author’s knowledge, there exists no graphical representation of the combination method, which can visually explain its assumptions, steps, and sensitivities to input parameters. The present short article offers such a graphical depiction in the hope that it be useful in the teaching and understanding of the methodology.

2. Diagram description

The construction of the combined diagram (CD) begins with the $(H_0, \lambda E_0)$ space shown in Fig. 1a, in which the axes correspond to surface sensible $(H_0)$ and latent heat fluxes $(\lambda E_0)$ with $\lambda$ the latent heat of water vaporization and $E_0$ the evaporative flux at the surface. The surface energy balance (SEB) restriction is

$$H_0 + \lambda E_0 = R_{N0} - G_0,$$  \hspace{1cm} (1)
where $R_{N0}$ is the net radiation flux reaching the surface and $G_0$ is the soil heat flux into the subsurface. The term $A_0 = R_{N0} - G_0$ will be called the surface available energy and will be treated as a known constant hereafter. The SEB restriction (1) defines in the CD the inclined line (labeled SEB) in Fig. 1a. Any point $(H_0, \lambda E_0)$ on the SEB line satisfies the surface energy balance for a given $A_0$, so that the surface flux partition problem cannot be fully determined solely with the SEB.

The second step is to convert the SEB line into a surface temperature axis. For this purpose we use a bulk transference equation for sensible heat in the form

$$H_0 = \rho c_p \frac{(T_0 - T_z)}{r_H},$$

(2)

where $T_0$ is the surface temperature, $T_z$ is the temperature measured at height $z$, $r_H$ is the resistance for heat transfer between the surface and height $z$, and $\rho$ and $c_p$ are the density and specific heat at constant pressure of the air, respectively. Equation (2) relates the surface temperature $T_0$ to the sensible heat flux:

$$T_0 = T_z + \frac{r_H}{\rho c_p} H_0,$$

(3)

for given $T_z$ and $r_H$. Since each point on the SEB line corresponds to a unique value of $H_0$, (3) shows that the SEB line can be graduated in terms of the corresponding surface temperature $T_0$, as shown in Fig. 1b. At the intersection of the SEB line with the $\lambda E_0$ axis (point A) the surface temperature has the value $T_z$, since at this point the surface energy balance requires $H_0 = 0$. At point B, on the other hand, $H_0 = A_0$, so that $T_0 = T_z + A_0 r_H/ (\rho c_p)$. The temperature interval between points A and B is thus $\Delta T = A_0 r_H/(\rho c_p)$. When interpreting the SEB line as a surface temperature axis, it will be called the $T_0$ axis.

The third step in the CD construction is to define a water vapor specific humidity axis ($q$ axis) perpendicular to the $T_0$ axis and passing through point A (Fig. 1c). The intersect of the $q$ axis with the $T_0$ axis is set at the value of the specific humidity at height $z$, $q_z$, for reasons that will become apparent shortly. The scaling of the $q$ axis, on the other hand, is defined such that the humidity at point C, located right above point B, has a value $q_C = q_z + \Delta q$, with $\Delta q = \gamma \Delta T$ and $\gamma = c_p/\lambda$ is the psychrometric constant.

Before completing the CD diagram we add two reference curves in the $(T_0, q)$ space. The first curve is the saturated water vapor specific humidity relation $q_{sat}(T_0)$ as shown in Fig. 2a. The intercept of the $q_{sat}(T_0)$ curve with the $q$ axis marks the saturation vapor deficit at the measurement height, $\delta q = q_{sat}(T_z) - q_z$, as shown in Fig. 2a.

The second reference curve is a transfer equation for water vapor, which can be written as

$$\lambda E_0 = \gamma (q_0 - q_z),$$

(4)

where $q_0$ is the surface humidity, and $r_V$ is the appropriate resistance for vapor transfer between the surface and height $z$. Equation (4) can be solved for $q_0$.
Since each point on the SEB line has a unique value of $lE_0$, Eq. (5) defines a straight line in the $(T_0, q)$ space (line labeled $q_0$ in Fig. 2b). This line always crosses the $T_0$ axis at point B, where $q_0 = q_z$ because at this point the SEB restriction requires $lE_0 = 0$. The slope of the $q_0$ line, however, depends on the $rV/rH$ ratio. In the particular case that $rV = rH$, it is easy to verify that the $q_0$ line is the straight line connecting points B and C as shown in Fig. 2b.

We have thus completed the basic CD. Since in the same plane we are considering $(H_0, \lambda E_0)$ and $(T_0, q)$ points, we shall use the convention that fine lines in the CD are to be interpreted in the $(H_0, \lambda E_0)$ space, while bold lines are to be interpreted in the $(T_0, q)$ space. The connection between both spaces is the SEB line or the $T_0$ axis.

3. Potential and nonpotential evaporation rates in the CD

a. Potential evaporation

For the particular case of a fully saturated surface that imposes no additional resistance to evaporation, then

$$r_V = r_H = r_a,$$  \hspace{1cm} (6)

and

$$q_0 = q_{sat}(T_0),$$  \hspace{1cm} (7)

where $r_a$ is the aerodynamic resistance for the transfer of heat and water between the surface and height $z$.  Condition (7) is found in the $(T_0, q)$ space at point P (see Fig. 3a) where the $q_{sat}$ curve intersects the $q_0$ line (the latter with the slope corresponding to the $r_V = r_H$ case). The associated potential latent heat flux, $\lambda E_p$, is found by translating the coordinates of P into the $(H_0, \lambda E_0)$ space passing through the SEB line (Fig. 3a). This graphical representation of $\lambda E_p$ shows very clearly all the conditions satisfied simultaneously by a combination potential evaporation rate—namely, the surface energy balance, the saturation of the surface, and the turbulent transfer relations for heat and vapor with the same aerodynamic resistances. Furthermore, it makes clear that this definition of potential evaporation has associated with it corresponding values of surface temperature and surface sensible heat flux (marked $T_{p0}$ and $H_{p0}$ in Fig. 3a).

b. Nonpotential evaporation: Surface resistance

Evaporation rates over vegetated surfaces are usually treated in the combination framework by assuming that plants introduce an additional resistance to the transfer of water vapor between the surface and the air, so that in this case $r_V > r_H$. In the CD this effect is illustrated in Fig. 3b, where the slope of the $q_0$ line has been increased because of the vegetation resistance. As a result, the actual surface latent heat flux, $\lambda E_{0,i}$ is less than $\lambda E_p$.

c. Nonpotential evaporation: Surface relative humidity and surface humidity deficit

Evaporation rates over nonsaturated soil are usually treated in the combination framework by assuming that
the surface has a relative humidity, RH$_0$, less than 100% or has a surface humidity deficit, $d q_0$, with respect to saturation. In the CD these approaches are illustrated in Figs. 3c,d, where the $q_{sat}$ has been affected by a factor RH$_0$ or subtracted and amount $d q_0$ in order to define the actual surface latent heat flux, $\lambda E_0$, which in both cases results lower than $\lambda E_p$.

d. Apparent potential evaporation

The apparent potential evaporation, $\lambda E_{pa}$, is that computed assuming that the surface is saturated at the actual temperature of the nonsaturated surface. Figure 4 shows its depiction in the CD. In Fig. 4a we show the actual evaporation, $\lambda E_0$, for a soil with a surface relative humidity less than 1 (see previous subsection). The saturated specific humidity at the actual surface temperature is at point P$''$. The apparent potential evaporation is found as shown in the figure by following the path P$''$-Q-R. This depiction of $\lambda E_{pa}$ in the CD shows that it is always larger or equal than $\lambda E_p$ and that as the surface dries and $\lambda E_0$ decreases the value of $\lambda E_{pa}$ increases rapidly, while $\lambda E_p$ maintains a constant value. More important, $\lambda E_{pa}$ does not satisfy the SEB and aerodynamic restrictions with the same surface temperature. Figure 4b, on the other hand, shows the $\lambda E_{pa}$ computed for a surface with a water vapor resistance greater than that for heat transfer.

4. Linear saturation curve expressions

A common simplification in the analysis is to linearize the saturation water vapor curve using the derivative $s = \partial q_{sat}/\partial T$ as the slope of the straight line. The CD with a linear saturation curve is shown in Fig. 5. The angle between the $q_{sat}$ line and the $T_0$ axis has in the CD a tangent equal to $s/\gamma$, because
With this linearization, obtaining expressions for actual and potential evaporation rates using the CD becomes a matter of careful geometrical analysis. For example, by doing the geometrical analysis of Figs. 6a–d, the following expressions can be derived:

\[
\lambda E_p = \frac{s}{s + \gamma} A_0 + \frac{\rho \alpha \delta_q}{s + \gamma} \frac{r_H}{r_H}
\]

(9)

\[
\lambda E_0 = \frac{s}{s + \gamma r_v/r_H} A_0 + \frac{\gamma r_v/r_H}{s + \gamma} \left(\frac{r_H}{r_H}\right) \frac{\rho \alpha \delta_q}{r_H}
\]

(10)

\[
\frac{E_0}{E_p} = \left(1 + \frac{\gamma}{s + \gamma} \frac{r_v}{r_H}\right)^{-1}, \text{ and}
\]

(11)

\[
\frac{E_0}{E_p_a} = \frac{r_H}{r_v}
\]

(12)

Equations (9)–(12) are equivalent to Eqs. (5.26), (5.35), (5.37), and (5.39) of Garratt (1994), respectively. In this textbook, however, they are derived from a manipulation of several mathematical equations, while with the CD the precise meaning of each term in the left-hand side of the equations can be directly seen and their right-hand sides result from geometrical arguments.

5. Discussion and conclusions

Some elements of the combination methodology are not included in the simple CD construction and usage presented above. The distinction between temperature and potential temperature differences across the atmospheric surface layer and the possibility of the transfer resistances and even the available surface energy to vary with surface temperature have not been considered in the analysis. These, however, can arguably be considered as secondary elements of the methodology, which can be addressed once its main assumptions and steps, graphically evident in the CD, are properly understood.

On the other hand, from the CD it is simple to infer how the values of \(\lambda E_p\) and \(\lambda E_0\) might change for different values of the available energy at the surface (translate the SEB line), for different values of the vapor deficit of the air (translate the \(q_{sat}\) curve), for different values of the...
aerodynamic resistance for vapor transfer (change the slope of the $q_0$ line), or of the surface relative humidity (affect the $q_{sat}$ curve by a constant factor). One sensitivity that is not as straightforward to show in the CD is the sensitivity of potential evaporation, $E_p$, to changes in the aerodynamic transfer resistance, $ra$. Because for potential evaporation $r_H = r_V = ra$, the $q_0$ line in the CD is locked in its vertical position and does not change with $ra$. However, by its definition the value of the scaling factor of the $q$ axis, $D_q$, increases with $r_H$, so that for a constant deficit $\delta_q$ an increase in $ra$ induces a translation of the $q_{sat}$ curve toward the $T_0$ axis. The result is a decrease in $E_p$ for an increase in $ra$, all other factors remaining constant.

An important concept associated to combination theory is that of equilibrium evaporation rate (e.g., Raupach 2001; Betts 1994). Following Garratt (1994), equilibrium evaporation rate is that computed over a saturated surface under advection-free conditions in which the humidity deficit in the surface layer is zero ($\delta_q = 0$). Figure 7 illustrates this situation in the CD. The Bowen ratio in this case is given by $\tan\beta = H_0/E_p$. Analysis of the figure shows that triangles ABC and CDE are geometrically similar, so that the angles $\alpha$ and $\alpha'$ are equal, as well as $\beta$ and $\beta'$. From the CD we know that $\tan\alpha = s/\gamma$, so that for equilibrium evaporation the Bowen ratio results as $\tan\beta = \gamma/s$. While this result is not new, the CD provides a new way of obtaining it.
As presented, the main action in the CD occurs in the $(T_0, q_0)$ space. The $(H_0, \lambda E_0)$ space, however, should be of help in relating the SEB to the dynamics of the ABL, as the vector $(H_0, \lambda E_0)$ is one important element in the water-temperature diagrams used by Betts (1992) to describe ABL budgets. Work along this line may be pursued in the future.

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REFERENCES


